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The Bitcoin Price Prediction by Vector Auto-Regression (VAR) Model

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ABSTRACT: The purpose of this paper is to assess the ability of a VAR model, used to predict. The results of the estimates lead to adopting a VAR model. However, the performances of this model are quite close, for certain horizons, to those performed by the forecasting organizations for the time series. We will first do a detailed analysis of Bitcoin prices, including the closing price. Next, we will move on to modeling the Bitcoin series using the VAR model, which will then be used for forecasting. We will move on to modeling the Bitcoin series using the VAR model, which consumers will then use.

Keywords: Bitcoin; Time Series; VAR.

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1. INTRODUCTION

This work is part of a larger study of the evaluation of forecasts according to Borowski and alii, (1991). Indeed, the study cited showed that the minimum standards made up of naïve forecasts, an identical reproduction of the last known value, were on average of a quality that was markedly lower than that of the forecasts from the Forecasting Department. We compare here the latter with a reference of purely statistical forecasts provided by the VAR modelling. The idea of comparing the forecasts of an institute with those provided by a VAR model is not new and there are many illustrations of it in the literature. Examples include McNees (1986), Zarnowitz (1986), Wallis (1989), Germain (1990), LeSage (1990). A broad consensus seems to emerge from this work: VAR modeling generally provides forecasts of a quality quite comparable, and sometimes even superior, to those of forecasting institutes. After recalling the methodology of VAR models (in particular the Johansen method), we compare the results obtained by simulating forecasts with these models with the results obtained by the Forecasting Department over the recent period. The virtual currency “Bitcoin” represents the digitization of an anonymous aspect which is characterized by its decentralization, i.e. the fact that no State or banking entity controls it. In this sense, Bitcoin has its own characteristics, the true identity of which remains unknown until today; we therefore refer to the pseudonym that has been left to the public “Satoshi Nakamoto”, creator of Bitcoin constant.

According Koop and Korobilis, (2009), the VAR model is an extension of the univariate autoregression model to multivariate time series data. In the VAR structure, each variable is a linear function of past lags of itself and past lags of other variables. However, the limited length of the data sets can produce over-parameterization problems. This paper focuses on forecasting Bitcoin prices using a Vector Auto-Regression (VAR) model, a popular approach for analyzing time series data with interdependent variables. It reviews previous studies that used models like ARIMA, GARCH, and machine learning, highlighting the VAR model's strength in capturing relationships between Bitcoin prices and external factors such as macroeconomic indicators and market sentiment. The methodology involves constructing the model, testing for stationarity, and validating with both in-sample and out-of-sample data. Results show the model's forecasting accuracy and its limitations during volatile market conditions. The conclusion emphasizes the VAR model's effectiveness in predicting short-term Bitcoin price movements and suggests future research directions.

2. LITERATURE REVIEW

2.1. Bitcoin price volatility

The paper is subdivided into three sections, namely: the use of the vector auto-regression (VAR) model.
According to Brito (2014), Bitcoin allows consumers for the first time to carry out person-to-person electronic transactions without the need for an intermediary between them, such as cash. Moreover, transactions made in the digital space with BTC allow individuals to push payments directly to merchants without having to share personally identifiable information, which could be intercepted by cybercriminals for fraud.

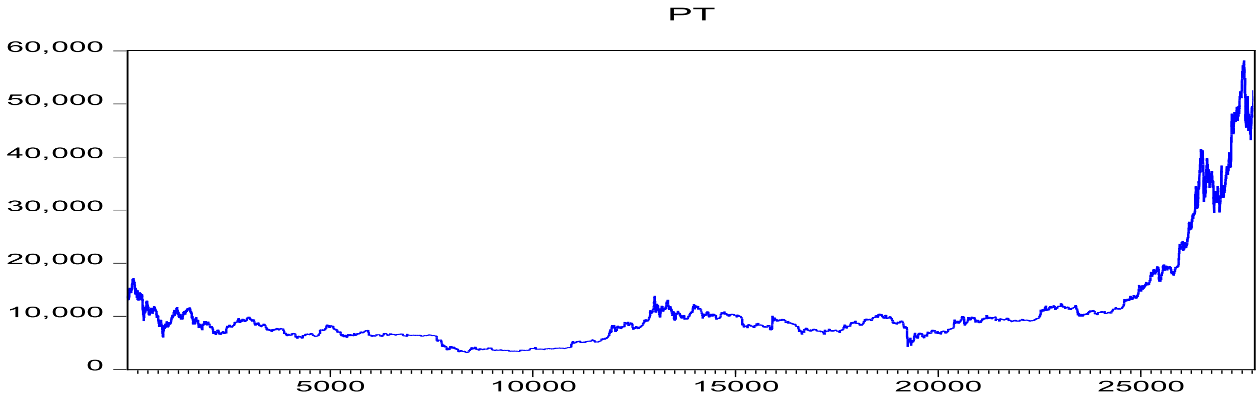
2.2. Time Series: The Price volatility Prediction

Volatility can be defined as a measure of the price dispersion of a financial asset. Market participants and investors are therefore interested in an accurate estimate of volatility in the cryptocurrency market.
This is the result of the correlation between volatility and investment returns. It should be noted that volatility is not directly observable and therefore there is a growing need for an efficient model that can capture price volatility in the cryptocurrency market. As bitcoin has gradually had a place in financial markets and portfolio management, time series analysis is a useful tool for studying the characteristics of bitcoin prices and returns, and extracting meaningful statistics to predict future values. from the Serie.

2.3. The price volatility Bitcoin to time series

This regular least squares (OLS) regression is a method for finding a linear relationship between two or more variables. To start, let's define a linear model as a function X, which equals Y with an error: $Y = \beta X + \varepsilon$; where Y is a dependent variable, X is an independent variable, ε is the magnitude of the error, and β is multiplier X. The OLS task is to print the value β in order to minimize ε.

Figure N°1: The price volatility Bitcoin to time series[1].



2.4. The VAR Model

The Vector Autoregressive (VAR) model is a statistical model developed by Christopher Sims in the early 1980s that captures the interdependencies between multiple time series.

- In a VAR model, the variables are treated symmetrically so that each of them is explained by its own past values and by the past values of the other variables.Autoregressive models (AR)

It is one of the methods used to model univariate time series data, where the current observed value is assumed to be a function of past values plus a random shock. The process $\{X_t\}$ is said to be autoregressive of order p, denoted AR (p) if,

$$X_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \quad \dots \dots \dots . . \varphi_p X_{t-p} + \varepsilon_t , \{\varepsilon_t\} \sim N(0, \sigma^2) \qquad (1)$$

$$X_t - \varphi_1 X_{t-1} - \varphi_2 X_{t-2} - \quad \dots \dots \dots . . \varphi_p X_{t-p} = \varepsilon_t$$

Or $(1- \varphi_1 L^1 - \dots - \varphi_P L^p)\, X_t = \varepsilon_t$

So an autoregressive model is simply a linear regression of the current value in the series against one or more previous values in the series. Therefore, we can easily determine current production, but the weakness of the autoregressive model is that past disturbances are not taken into account.

2.5. VAR-Cointegration of BTC price predictions

The cointegration is a statistical property of time series introduced in economic analysis, notably by Engel and Newbold (1974). In simple terms, Cointegration allows to detect the long-term relationship between two or more time series. We call two series are integrated if it is possible to express a linear combination of the two series which will be integrated of lower order. The Johansen's test we see that the series cointegrate more than a 95% threshold. Thus there are at least five cointegrating vectors when operated on the non-stationary series, making them stationary. A vector autoregression (VAR) model (Sims 1993), (Kuschnig et al. 2020) and (Kuschnig and Vashold,2019) was developed to understand the relationship between the system of variables of interest, and capture the interdependence between series. Thus, the

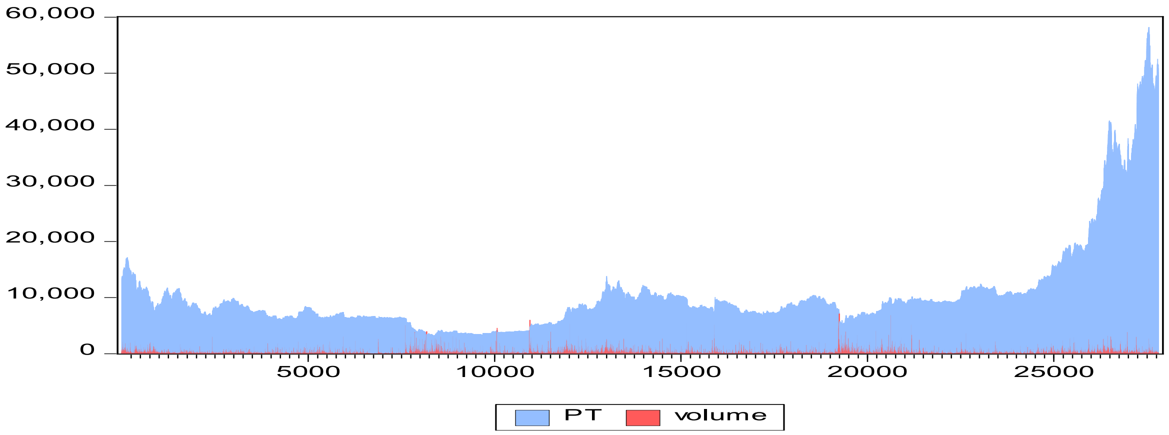
[1] https://newdaycrypto.com/fr/supply-and-demand-model-for-bitcoin-price/

VAR is presented through the following formula:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \ldots + \beta_n Y_{t-n} + \beta_{n+1} X_t + \epsilon_t$$

Where the β are vectors of constants and coefficients representative of the relationship between the variables, and n is the number of lags used in the VAR model.

Figure N° 2: Cointegrated Series [2]



3. METHODOLOGY AND MODELING OF A TIME SERIES

3.1. Box & Jenkins Model (1976)

The Box & Jenkins (1976) model is used to determine an adequate methodology for showing a chronicle for the purpose of predicting nearby eventual values. Indeed, the objective of this methodology is the modeling of a time series according to its past and present values in order to determine the appropriate **ARIMA** process by principle of parsimony. This methodology suggests model identification with a three-step procedure including model estimation and model validation. Then, the three steps are identified as the sequences :

3.1.1. Unit root test

Whether the time series is stationary or not is a very important concept before drawing conclusions in time series analyses. Therefore, Augmented Dickey Fuller (ADF), Phillips Perron (PP), and Kwiathowsky-Phillips-Schmidt-Shin (KPSS) tests were used to verify the stationery of the series. The test is based on the assumption that a series of time data Y_t follows a random movement: $Y_t = \rho\, Y_{t-1} + e_t$

Where ρ is the characteristic root of an AR polynomial and e_t is a purely random process with mean zero and variance $\mathbf{Var}\ (e_t)$ equal to zero..

[2] CAF AIMANE (2021). 'Bitcoin Price Prediction Using Vector Auto-Regression (VAR) Model; Aix-Marseille School Of Economics, Marseille, Frensh

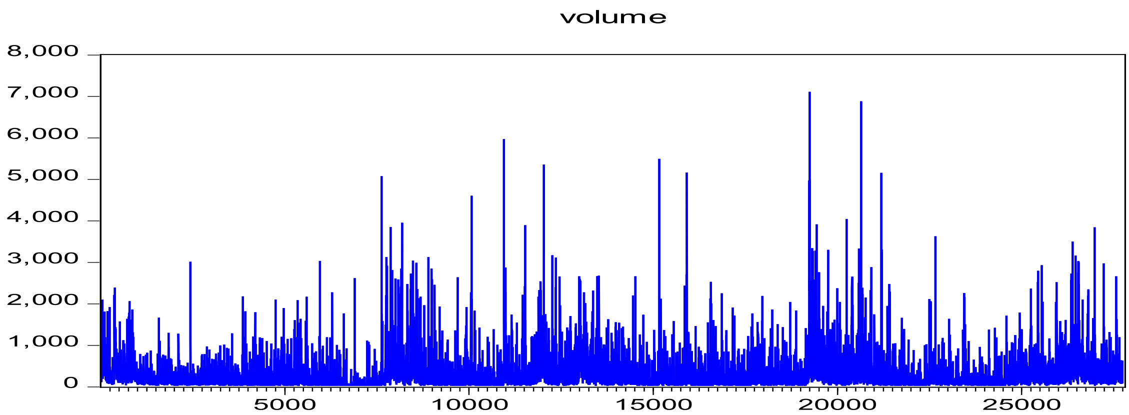
3.1.2. Augmented Dickey Fuller Test (ADF)

The Augmented Dickey Fuller test (1979) is a root test of the **ADF** unit, which therefore tests the acceptance, or rejection of one of the following two hypotheses:

H_0: $\rho = 1$ non stationary or else H_1: $\rho \neq 1$ stationary.

The **ARIMA** model is the course of the Box and Jenkins method which makes it possible to determine a time series according to its characteristics. It consists of several steps:

Figure Nº 3: The process of creating a dynamic forecast from VAR[3]



4. RESULT AND DISCUSSION

4.1. Result

4.1.1. Descriptive statistics

Table N°1: Descriptive analyzes

	PT	VOLUME
Mean	10371.57	253.8413
Median	8558.150	155.6579
Maximum	58179.70	7113.421
Minimum	3141.400	0.000000
Std. Dev.	8208.034	331.9514
Skewness	3.212474	5.201308
Kurtosis	14.14600	52.57350
Jarque-Bera	191567.7	2969637.
Probability	0.000000	0.000000
Sum	2.88E+08	7051205.
Sum Sq. Dev.	1.87E+12	3.06E+09
Observations	27778	27778

Source: Made by the authors

[3] https://newdaycrypto.com/fr/supply-and-demand-model-for-bitcoin-price

4.1.2. Unit root test

➢ *Augmented DICKEY-FULLER*

Table Nº 2: Unit root test

Null Hypothesis: RT has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 60 (Automatic - based on SIC, maxlag=63)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-36.09636	0.0000
Test critical values:	1% level	-3.958148	
	5% level	-3.409858	
	10% level	-3.126636	

*MacKinnon (1996) one-sided p-values.

➢ *Augmented Dickey-Fuller Test Equation*

Table Nº 3: Augmented Dickey-Fuller Test Equation

Dependent Variable: D(RT)
Method: Least Squares
Date: 08/16/21 Time: 23:13
Sample (adjusted): 1/02/2012 21:00 3/04/2021 08:00
Included observations: 79862 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RT(-1)	-1.079694	0.029911	-36.09636	0.0000
D(RT(-1))	0.024066	0.029657	0.811481	0.4171
D(RT(-2))	-0.008246	0.029399	-0.280488	0.7791
D(RT(-3))	-0.003314	0.029122	-0.113804	0.9094
D(RT(-4))	-0.036614	0.028822	-1.270358	0.2040
D(RT(-5))	-0.049564	0.028506	-1.738749	0.0821
D(RT(-6))	-0.045615	0.028191	-1.618058	0.1057
D(RT(-7))	-0.059954	0.027882	-2.150304	0.0315
D(RT(-8))	-0.025205	0.027567	-0.914342	0.3605
D(RT(-9))	-0.030781	0.027253	-1.129466	0.2587
D(RT(-10))	-0.006852	0.026940	-0.254350	0.7992
D(RT(-11))	0.007393	0.026636	0.277552	0.7814
D(RT(-12))	0.046558	0.026342	1.767445	0.0772
D(RT(-13))	0.099138	0.026054	3.805109	0.0001
D(RT(-14))	0.091216	0.025778	3.538596	0.0004
D(RT(-15))	0.108741	0.025488	4.266408	0.0000
D(RT(-16))	0.137236	0.025195	5.446866	0.0000
D(RT(-17))	0.112267	0.024904	4.508088	0.0000
D(RT(-18))	0.110826	0.024611	4.503179	0.0000
D(RT(-19))	0.106210	0.024322	4.366753	0.0000
D(RT(-20))	0.097860	0.024047	4.069510	0.0000
D(RT(-21))	0.097822	0.023764	4.116473	0.0000
D(RT(-22))	0.072814	0.023478	3.101313	0.0019
D(RT(-23))	0.068475	0.023186	2.953242	0.0031

D(RT(-24))	0.018507	0.022882	0.808790	0.4186
D(RT(-25))	-0.018405	0.022559	-0.815871	0.4146
D(RT(-26))	-0.015118	0.022240	-0.679777	0.4966
D(RT(-27))	-0.038442	0.021895	-1.755739	0.0791
D(RT(-28))	-0.029925	0.021552	-1.388484	0.1650
D(RT(-29))	-0.024749	0.021203	-1.167257	0.2431
D(RT(-30))	-0.035242	0.020832	-1.691699	0.0907
D(RT(-31))	-0.036318	0.020458	-1.775303	0.0759
D(RT(-32))	-0.012040	0.020083	-0.599501	0.5488
D(RT(-33))	-0.008522	0.019694	-0.432697	0.6652
D(RT(-34))	-0.018445	0.019293	-0.956013	0.3391
D(RT(-35))	0.016405	0.018903	0.867866	0.3855
D(RT(-36))	0.004581	0.018502	0.247562	0.8045
D(RT(-37))	0.023526	0.018121	1.298248	0.1942
D(RT(-38))	0.034980	0.017770	1.968516	0.0490
D(RT(-39))	0.036040	0.017416	2.069397	0.0385
D(RT(-40))	0.030598	0.017073	1.792160	0.0731
D(RT(-41))	0.033940	0.016723	2.029549	0.0424
D(RT(-42))	0.010018	0.016369	0.612004	0.5405
D(RT(-43))	-0.003101	0.016008	-0.193741	0.8464
D(RT(-44))	-0.010893	0.015636	-0.696662	0.4860
D(RT(-45))	-0.015174	0.015274	-0.993419	0.3205
D(RT(-46))	-0.016522	0.014877	-1.110564	0.2668
D(RT(-47))	0.002949	0.014452	0.204087	0.8383
D(RT(-48))	-0.019374	0.014019	-1.381961	0.1670
D(RT(-49))	-0.034231	0.013526	-2.530740	0.0114
D(RT(-50))	-0.060694	0.012972	-4.678822	0.0000
D(RT(-51))	-0.081525	0.012375	-6.587910	0.0000
D(RT(-52))	-0.090287	0.011719	-7.704408	0.0000
D(RT(-53))	-0.104332	0.011035	-9.454714	0.0000
D(RT(-54))	-0.103787	0.010263	-10.11267	0.0000
D(RT(-55))	-0.118098	0.009447	-12.50054	0.0000
D(RT(-56))	-0.124545	0.008547	-14.57200	0.0000
D(RT(-57))	-0.098673	0.007565	-13.04250	0.0000
D(RT(-58))	-0.057949	0.006504	-8.909401	0.0000
D(RT(-59))	-0.021121	0.005209	-4.054455	0.0001
D(RT(-60))	-0.020790	0.003575	-5.816039	0.0000
C	-1.029509	0.622059	-1.655000	0.0979
@TREND("12/31/2011 12:00")	4.28E-05	1.34E-05	3.182989	0.0015

R-squared	0.537151	Mean dependent var	0.002505
Adjusted R-squared	0.536792	S.D. dependent var	129.0876
S.E. of regression	87.85627	Akaike info criterion	11.79007
Sum squared resid	6.16E+08	Schwarz criterion	11.79740
Log likelihood	-470726.3	Hannan-Quinn criter.	11.79232
F-statistic	1493.700	Durbin-Watson stat	2.000097
Prob(F-statistic)	0.000000		

Source: Made by the authors

➢ *PHILIP PERRON :*

Table N°4*:* PHILIP PERRON

Null Hypothesis: RT has a unit root

Exogenous: Constant, Linear Trend

Bandwidth: 72 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-299.1839	0.0001
Test critical values:	1% level	-3.958147	
	5% level	-3.409857	
	10% level	-3.126636	

*MacKinnon (1996) one-sided p-values.

Residual variance (no correction)	7883.472
HAC corrected variance (Bartlett kernel)	7380.190

Source : Made by the authors

➢ *Phillips-Perron Test Equation*

Table N°5*:* Phillips-Perron Test Equation

Dependent Variable: D(RT)

Method: Least Squares

Date: 08/16/21 Time: 23:20

Sample: 12/31/2011 12:00 3/04/2021 08:00

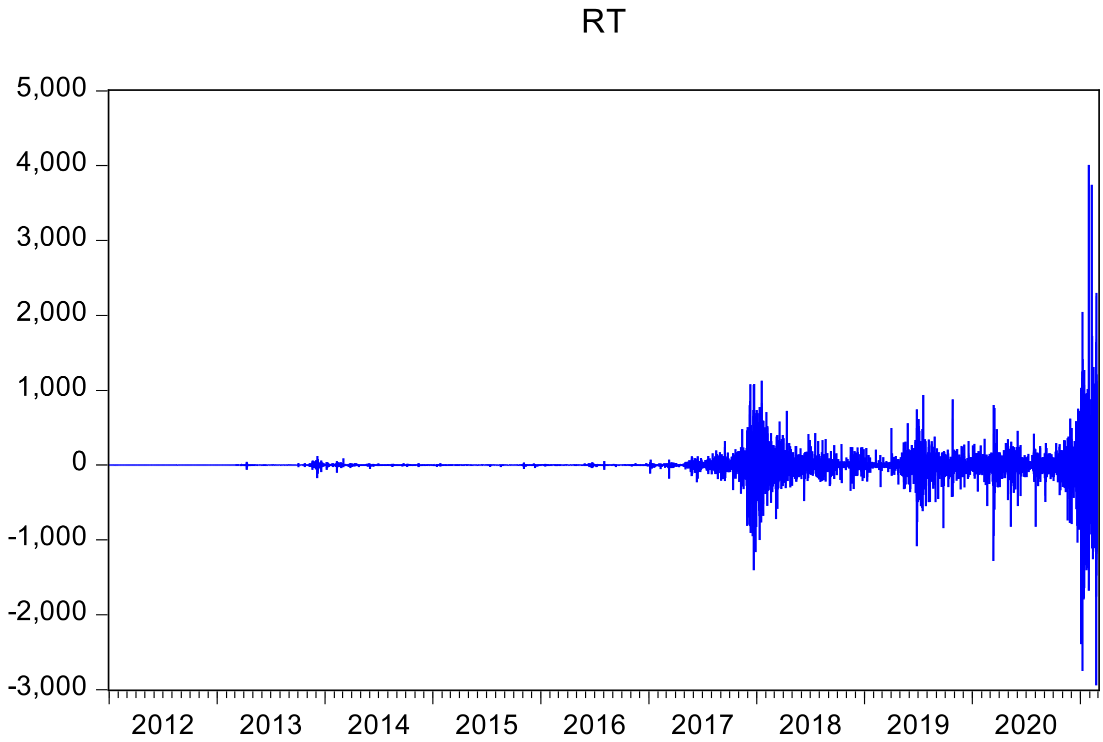
Included observations: 80279

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RT(-1)	-1.052371	0.003525	-298.5102	0.0000
C	-0.987453	0.626657	-1.575746	0.1151
@TREND("12/31/2011 12:00")	4.10E-05	1.35E-05	3.036018	0.0024

R-squared	0.526072	Mean dependent var	0.006068
Adjusted R-squared	0.526060	S.D. dependent var	128.9749
S.E. of regression	88.79058	Akaike info criterion	11.81048
Sum squared resid	6.33E+08	Schwarz criterion	11.81082
Log likelihood	-474063.6	Hannan-Quinn criter.	11.81058
F-statistic	44554.17	Durbin-Watson stat	2.002016
Prob(F-statistic)	0.000000		

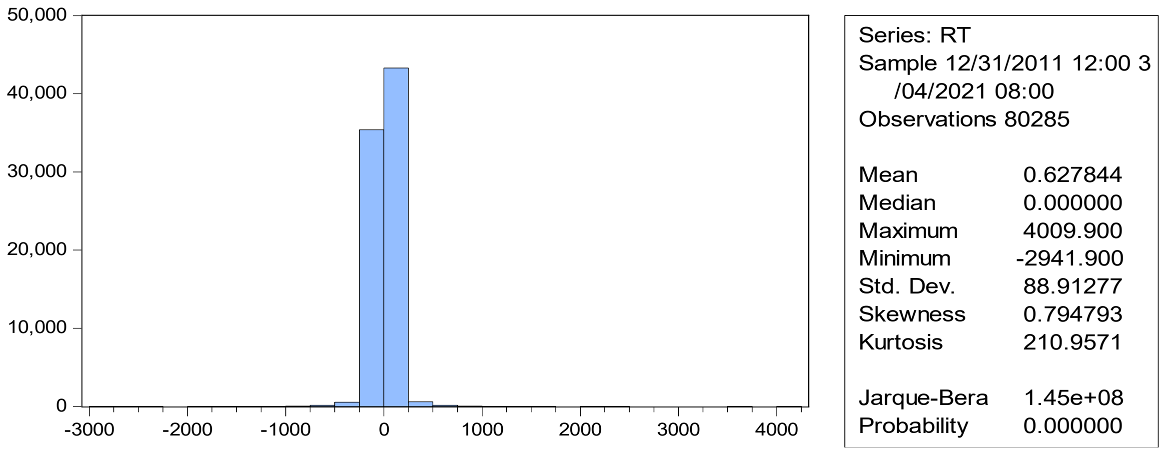
Source : Made bv theAuthors

Figure Nº 4: Total Return



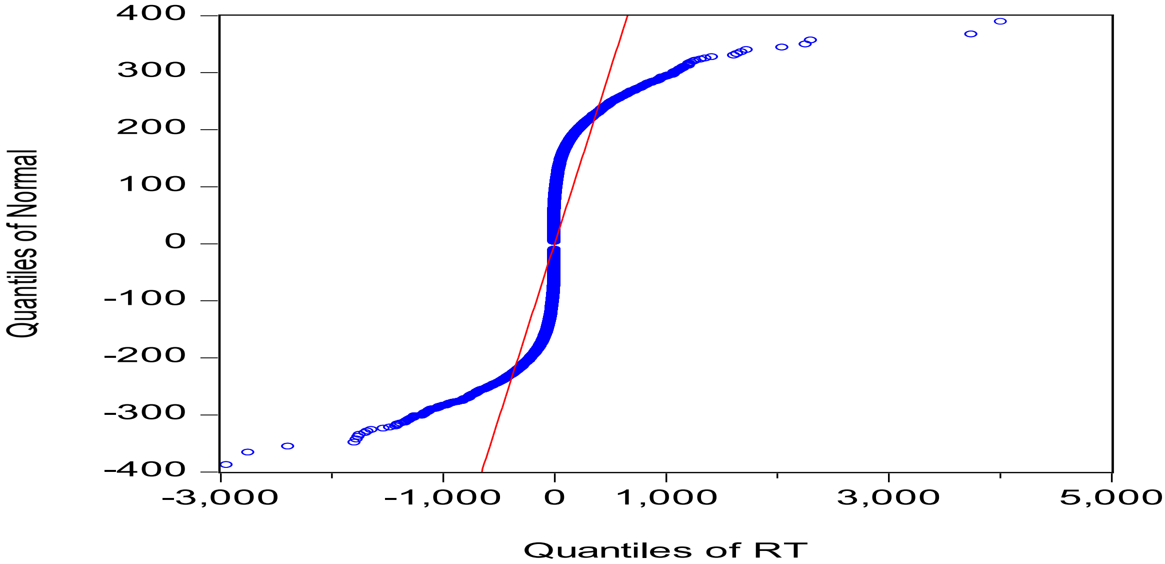
Source : Made by the authors

Figure 5: Normality Test



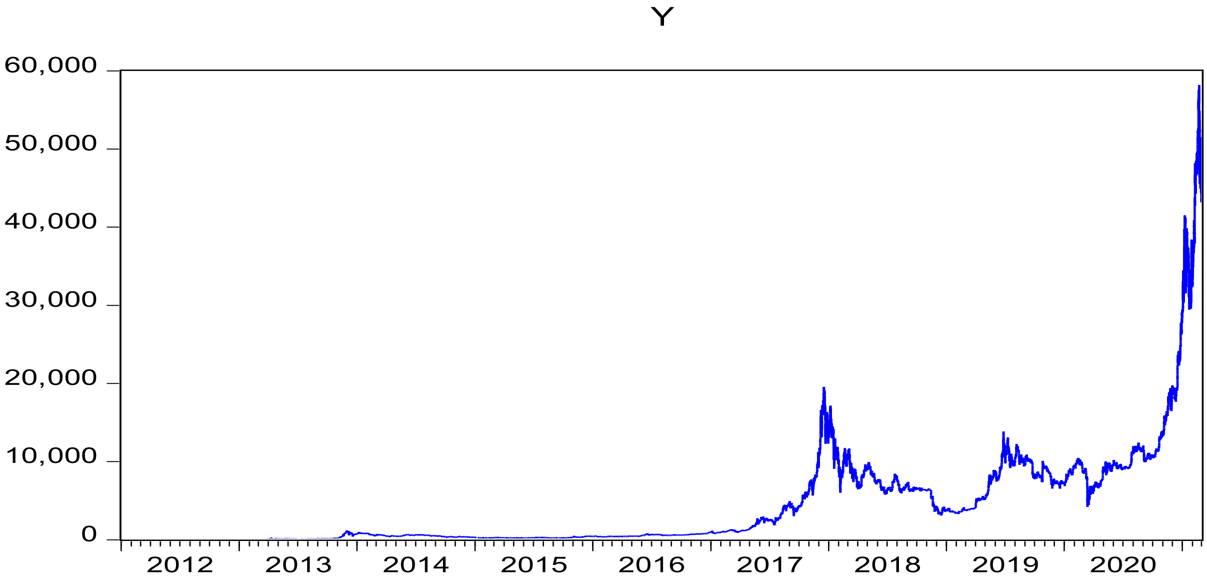
Source : Made by the authors

Figure 6: Quantiles of Total Return



Source : Made by the authors

Figure 7: Trend PT (likely "Price") and VOLUME (Volume traded) bitcoin price



Source : Made by the authors

Table N° 6*:* Autocorrelation AC and PAC

Date: 09/23/22 Time: 12:37
Sample: 1 27801
Included observations: 27778
Correlations are asymptotically consistent approximations

PT,VOLUME(-i)	PT,VOLUME(+i)	i	lag	lead
		0	0.0839	0.0839
		1	0.0843	0.0845
		2	0.0845	0.0848
		3	0.0848	0.0849
		4	0.0850	0.0853
		5	0.0851	0.0854
		6	0.0850	0.0855
		7	0.0852	0.0855
		8	0.0852	0.0855
		9	0.0852	0.0856
		10	0.0852	0.0857

Source : Made by the authors

Table N°. 7: Unit root test
Group unit root test: Summary
Series: PT, VOLUME
Date: 09/23/22 Time: 12:39
Sample: 1 27801
Exogenous variables: Individual effects
Automatic selection of maximum lags
Automatic lag length selection based on SIC: 23 to 25
Newey-West automatic bandwidth selection and Bartlett kernel

Method	Statistic	Prob.**	Cross-sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	6.28373	1.0000	2	55399
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-8.65572	0.0000	2	55399
ADF - Fisher Chi-square	186.013	0.0000	2	55399
PP - Fisher Chi-square	18.4207	0.0010	2	55572

** Probabilities for Fisher tests are computed using an asymptotic Chi -square distribution. All other tests assume asymptotic normality.

Source : Made by the authors

➢ *Estimation VAR*

Vector Autoregression Estimates
Date: 09/23/22 Time: 13:19
Sample (adjusted): 3 27801
Included observations: 27766 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	PT	VOLUME
PT(-1)	0.951992	-0.010452
	(0.00600)	(0.01169)
	[158.537]	[-0.89447]
PT(-2)	0.048283	0.011851
	(0.00601)	(0.01169)
	[8.03725]	[1.01372]
VOLUME(-1)	0.011429	0.473600
	(0.00307)	(0.00597)
	[3.72773]	[79.3777]
VOLUME(-2)	-0.004335	0.122517
	(0.00306)	(0.00596)
	[-1.41432]	[20.5425]
C	-3.255059	88.05442
	(1.51633)	(2.95072)
	[-2.14667]	[29.8417]
R-squared	0.999701	0.307148
Adj. R-squared	0.999701	0.307048
Sum sq. resids	5.60E+08	2.12E+09
S.E. equation	142.0219	276.3690
F-statistic	23185826	3076.673
Log likelihood	-177003.5	-195488.9
Akaike AIC	12.75002	14.08153
Schwarz SC	12.75150	14.08301
Mean dependent	10372.43	253.9045
S.D. dependent	8209.425	331.9997
Determinant resid covariance (dof adj.)		1.54E+09
Determinant resid covariance		1.54E+09
Log likelihood		-372449.8
Akaike information criterion		26.82848
Schwarz criterion		26.83144
Number of coefficients		10

Source : Made bv the authors

4.2. Interpretations and discussions

The images provided display statistical results from a Vector Auto-Regression (VAR) model and group unit root test summaries. Here's an interpretation and discussion based on the visual content of the two outputs:

❖ *VAR Results Interpretation:*
- Dependent Variables: The two columns represent the dependent variables in the system: PT (likely "Price") and VOLUME (Volume traded). Each column reflects how current values of PT and VOLUME are influenced by their past values.

❖ *Lags*:
- PT(-1) and PT(-2) represent the first and second lags of the Price variable.
- PT(-1) has a highly significant positive effect on PT (0.9519, t-stat: 158.5). This indicates strong persistence in price changes, meaning the previous period's price is a strong predictor of the current period's price.
- PT(-2) has a much smaller effect (0.0483) but is still positive and significant.
- VOLUME(-1) has a positive significant effect on PT (0.0114, t-stat: 3.72), suggesting that previous volume affects current price, though the magnitude is smaller compared to PT
- VOLUME(-2) has a small negative but insignificant effect on PT.
- PT(-1) and PT(-2) have no significant effects on Volume.
- VOLUME(-1) significantly influences the current volume (0.4736, t-stat: 79.38), indicating strong persistence in trading volume.

❖ *Constant (C):*
- For PT, the constant is negative and significant (-3.25, t-stat: -2.14), suggesting that in the absence of other factors, the PT would decline.
- For VOLUME, the constant is large and significant (88.05, t-stat: 29.84), indicating a strong baseline for trading volume.

❖ *R-Squared:*
- PT's R-squared is extremely high (0.9997), showing that the model explains nearly all the variance in prices. This might indicate overfitting, or that past prices are extremely strong predictors of current prices.
- VOLUME's R-squared is much lower (0.3071), suggesting that the model does not explain the variability in trading volume as well as it does for price.

❖ *Diagnostic Statistics*:
- The Akaike Information Criterion (AIC) and Schwarz Criterion (SC) are given for model selection, with lower values indicating a better fit.
- Both equations show a high F-statistic, indicating the overall model is highly significant.

4.3. Group Unit Root Test

The group unit root test examines whether the PT and VOLUME time series are stationary.

Levin, Lin & Chu t-stat: This test assumes a common unit root process. The test statistic is 6.2837 with a probability of 1.000, indicating that the null hypothesis (of a unit root) cannot be rejected for this method.

Im, Pesaran, and Shin W-stat: Assumes individual unit root processes. The test statistic is -8.6557, and the probability is 0.000, which means the null hypothesis of a unit root is rejected, indicating stationarity.

ADF - Fisher Chi-square and PP - Fisher Chi-square also reject the unit root hypothesis, supporting the stationarity of the series.

❖ *Discussion*:

- Persistence of Price Movements: The VAR model shows that Bitcoin prices are strongly autocorrelated, meaning past prices are a strong predictor of current prices. This suggests a high level of momentum in the Bitcoin market.
- Impact of Volume: Volume has a smaller but still positive influence on price movements, which could indicate that trading volume does have some predictive power, though price history is far more influential.
- Stationarity: The unit root test results are mixed, but generally, the series (PT and VOLUME) appear to be stationary based on the Im, Pesaran, and Shin W-stat, ADF, and PP tests. This supports the use of VAR, which requires stationary data.

5. Conclusions

This study effectively utilized the Vector Auto-Regression (VAR) model to predict Bitcoin prices, revealing strong price persistence, with past prices significantly influencing current price movements. Trading volume, while statistically significant, played a smaller role compared to historical price data. The high R-squared for price indicates the model's accuracy in short-term forecasting, though the lower R-squared for volume suggests the need to incorporate additional external factors.

The unit root tests confirmed the stationarity of the time series, validating the model's robustness. However, the VAR model faces limitations during periods of high market volatility. Future research should consider integrating more variables such as macroeconomic data and market sentiment to improve prediction accuracy.

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